Abstract

In order to predict dynamic water processes in the soil under field conditions, it is necessary to collect a large amount of information, making research expensive and time-consuming, leading some researchers to utilize mathematical models developed to accurately describe the hydrodynamic characterization of soils. The objective of this study was to test the sensitivity of Hydrus-1D to variations in input information, and to determine which items need to be measured with greater accuracy. For this, the moisture and matric potential for a Yellow Latosol were evaluated through simulations. The systematic sensitivity analyses showed that the model was not very sensitive to variations in saturated hydraulic conductivity (Ks) and residual moisture ($\theta_r$). For the parameter $a$ (empirical), the values obtained indicated that the model has low to intermediate sensitivity. For the parameter $n$ (empirical dimensionless), the simulated values also presented low to intermediate sensitivity. For saturated moisture ($q_s$), the model presented relative sensitivity values from intermediate to high.

Keywords: mathematical modeling, matric potential, soil moisture.

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Resumo
Para realizar a predição dos processos da dinâmica da água no solo em condições de campo há a necessidade de coletar grande número de informações, o que torna a pesquisa onerosa e demorada, levando alguns pesquisadores a utilizarem modelos matemáticos desenvolvidos para descrever com propriedade e acuidade a caracterização hidrodinâmica de diferentes solos. Portanto, objetivou-se testar a sensibilidade do Hydrus-1D às variações das informações de entrada, e determinar quais deveriam ser aferidas com maior precisão. Para isso, através de simulações, foram avaliados o potencial matricial e a umidade para um Latossolo Amarelo. As análises sistemáticas da sensibilidade mostraram que o modelo foi pouco sensível às variações da condutividade hidráulica saturada (Ks) e da umidade residual ($\theta_r$). Para o parâmetro $a$ (empírico), os valores obtidos apontaram que o modelo tem sensibilidade de baixa a intermediária. Com relação ao parâmetro $n$ (empírico adimensional), os valores simulados também apresentaram sensibilidades de baixa a intermediária. Já para a umidade saturada ($q_s$), o modelo apresentou valores de sensibilidade relativa de intermediária a alta.

Palavras chave: modelagem matemática, potencial matricial, umidade do solo.

Introdução
Analysis and retrieval of hydrodynamic data has become an important subject in recent work, mainly because it is a necessary information source on which are based mathematical tools that can predict future scenarios. Historically, it has been observed that the use and implementation of modeling has increased (Soares, 2018), considering the number of studies that made use of some type of modeling (Martins Gomes et al., 2007, Oliveira et al., 2010, Soares et al., 2016).

Hillel (1998) states that in order to predict the dynamics of water and soil salts in field conditions, the following parameters must be known: the relationship between the matric potential (h) and soil volumetric moisture q (h) (soil water retention curve) and the relationship between hydraulic conductivity and volumetric moisture K (q), known as the hydraulic conductivity curve. However, this requires the collection of a large amount of information, making research expensive and time-consuming, leading some researchers to instead use mathematical models developed to accurately describe the hydrodynamic characterization of different soils and at various points in a faster and more reliable way, allowing for extrapolation of the data (Rivera et al., 2008; Castro et al., 2010).

Lima et al. (2001) state that, whenever possible, it is important to confirm the values collected with discretized values derived from the use of modeling, given the difficulty of obtaining data with precision, quality, and quantity necessary to perform the desired study.

The first simplified representations of water (and solutes) transport in the soil were made to provide rough estimates of the leaching of soluble materials. Following this, more complex models were developed to try to integrate the physical and chemical mechanisms that influence the movement of solutes. These models are usually based on solutions that use numerical
methods and are based on the predicted flow of water as a starting point for simulating the flow of solutes (Wagenet, 1986).

Many numerical models have emerged in recent decades. Among them, the following models can be cited: SiB (Simple Biosphere Model - Sellers et al., 1986), ISBA (Interaction Soil-Biosphere-Atmosphere - Noilhan & Planton, 1989), SiSPAT Antonino (1992), LAPS (Land Air Surface Scheme - Mihailović, 1996), BATS (Biosphere Atmosphere Transfer Scheme - Yang & Dickinson, 1996), PLATIN (Plant-Atmosphere Interaction - Grünhage & Haenel, 1997), and ALSIS (Atmosphere-Land-Surface Scheme - Irannejad & Shao, 1998). In general, they predict the transfer of water (and solutes) between the soil surface and the water table. It is worth mentioning that each model presents differences in the numerical solution to the Richards equation. Therefore, it is important to identify how the input parameters interfere in the modeling of soil water dynamics.

Each model is distinguished by its ability to simulate physical processes of the soil, considering or not the transport of water, heat, and solutes concomitantly. The Hydrus-1D model stands out by simulating such phenomena simultaneously. This model has been used in many scientific studies, principally in the simulation of soil water dynamics (Šimůnek; Valocchi, 2002; Hilner et al., 2008; Nimmer et al., 2010; al., 2015).

Pinho & Miranda (2014) simulated the vertical movement of water and potassium in unsaturated soils (Red-Yellow Latosol and Red Nitosol) using Hydrus-1D. These authors concluded that the model was a good tool to predict potassium displacement and the soil moisture profile.

Gonçalves et al. (2007) evaluated the efficiency of the Hydrus-1D model in predicting the water and salt content in alluvial soil, irrigated with water of varying quality. They observed that the Hydrus-1D model was able to predict the soil water content as well as the effects of irrigation on alluvial soil geochemistry, with a remarkable correspondence between the results observed in the field and the simulation. These authors emphasized this tool’s capacity for extrapolation and the conditions it brings together to predict the effects of irrigation water quality on soil and aquifers and on the implementation of good practices for irrigation and fertilization, helping to control and prevent environmental problems.

Although the Hydrus-1D model has been widely used to analyze soil hydrodynamics, it still lacks sensitivity testing. Whenever possible, these analyses should be performed, because it is possible for errors generated by the model to be introduced when using data with small variations inherent to instrumentation errors and equipment calibration (Zhou et al., 2012; Chen et al. 2014; Han et al., 2015).

According to Evangelista and Ferreira (2006), for simple models it is possible to obtain outputs as a function of the input data, demonstrating the sensitivity of the model as explicit functions.
However, as models become more complex, sensitivity is expressed more easily in the form of relative changes, graphs and tables, rather than functions. Sensitivity analysis has been widely used in assessing the reliability of input data in several areas of research (Pinho, 2009).

This study therefore sought to assess the sensitivity of the Hydrus-1D model to variations in input data used in the modeling of soil water dynamics, making it possible to identify which parameters need to be measured with greater precision in order to reduce the propagation of errors generated by numerical procedures.

Methodology

Study area and data acquisition
The study was carried out using data collected in the city of Areia, Paraíba, Brazil, 136 km from the state capital of João Pessoa. The data collection area is located at the geographical coordinates 06° 57' 48'' south latitude and 35° 41' 30'' west longitude, having an altitude of 618 m. Soil moisture was measured from the soil surface down to a depth of 0.13 m. At this depth, 90% of the root system could be found.

Soil moisture measurements were performed using thermocouples (108-L, Campbell Scientific, Inc., Logan, Utah, USA) and moisture sensors (CS 615, Campbell Scientific, Inc., Logan, Utah, USA) and recorded every half-hour for 72 hours by a datalogger (CR 10x, Campbell Scientific, Inc., Logan, Utah, USA).

The climate in the region, according to the Köppen classification, is of type As’ (hot and humid), with a rainy season during the fall-winter period, with heaviest rainfall in June and July (Brazil, 1972). The soil of the area is classified, according to EMBRAPA (2006), as Yellow Latosol. The layer of soil between the surface and a depth of 40 cm is texturally classified as Light Sandy Clay and the soil layer between 40 and 80 cm is classified as Sandy Clay (Lima, 2004).

About the HYDRUS-1D
The Hydrus-1D program was developed by Šimůnek and collaborators to perform water and solute flow analysis on completely saturated, unsaturated, or partially saturated porous media using the numerical solution of the Richards equation (Šimůnek et al., 1998; Šimůnek et al., 2008). This flow can be measured in several directions: vertical, horizontal, or with some degree of inclination.

To measure water transfer in the soil, the model solves the modified Richards equation (Equation 1).

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} [K (\frac{\partial h}{\partial z})] - S$$

Equation (1)
where \( h \) is the matric potential (cm), \( q \) is the soil volumetric moisture (cm\(^3\) cm\(^{-3}\)), \( t \) is the time (h), \( K \) is the hydraulic conductivity (cm h\(^{-1}\)), \( z \) is the special coordinate (cm), and \( S \) is the water extraction term (cm\(^3\) cm\(^{-3}\) h\(^{-1}\)) (Silva et al., 2015).

The simulation profile used had a depth of 0.13 m and was subdivided into equal layers of 0.01 m. This depth was chosen because it contains more than 90% of the root system of a bean crop already present at the locale. Below this depth, interest in the hydrodynamic processes decreases, as it would not be important for the existing crops.

Eleven observation nodes were selected at depths of 0.02 to 0.12 m. Depths of 0.01 and 0.13 m were used as upper and lower boundary conditions, respectively.

The van Genuchten-Mualem model (Van Genuchten, 1980a) was used to simulate the one-dimensional flow of water in the specified soil, considering a constant flow for both upper and lower boundary conditions (Equations 2 and 3). An initial time of 0 h and a final time of 72 h were used, divided into 144 equal sections, with an initial time step of 0.024 h, a minimum step of 0.00024 h, and a maximum step of 72 h. For the soil profile, 13 pressure values were used as the initial conditions, one for each 0.01 m of depth in the soil profile (Table 1). These values of soil matric potential were obtained from the Van Genuchten equation, using the moisture data determined by the sensor.

\[
\theta(h) = \begin{cases} 
\theta_r + \frac{\theta_s - \theta_r}{(1+|ah|^m)^m} & h < 0 \\
\theta_s & h \geq 0 
\end{cases} 
\]

**Equation (2)**
\[
K(\theta) = \begin{cases} 
    K_s \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^\lambda \left[ 1 - \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^\frac{1}{m} \right]^{m+2} & h < 0 \\
    K_s & h \geq 0 
\end{cases} \quad \text{Equation (3)}
\]

where \( q \) is the soil volumetric moisture (cm\(^3\)cm\(^{-3}\)), \( h \) is the matric potential (cm), \( K \) is the hydraulic conductivity (cm h\(^{-1}\)), \( \theta_r \) is the residual moisture (cm\(^3\)cm\(^{-3}\)), \( \theta_s \) is the saturated moisture (cm\(^3\)cm\(^{-3}\)), \( \alpha \) (cm\(^{-1}\)), \( n \) and \( m \) are dimensionless empirical parameters, where \( m = 1 - 1/n \), \( K_s \) is the hydraulic conductivity of saturated soil (cm d\(^{-1}\)), and \( l \) is an empirical value related to soil porosity.

**Sensitivity Analysis**

Five soil parameters (properties) were used in the sensitivity analysis: residual water content (\( \theta_r \)), saturated water content (\( \theta_s \)), hydraulic conductivity of saturated soil (\( K_s \)), and the empirical parameters \( n \) and \( \alpha \) of the van Genuchten-Mualem model (Genuchten, 1980b). Although the residual water content is considered null in some studies, it is necessary to evaluate the influence of this parameter on the model and, therefore, to decide whether it is necessary to maintain or remove it.

The systematic sensitivity analysis consisted of varying one of the five parameters in increments and decrements of 10 and 20%, while keeping the others fixed (initial values) (Table 2). The initial values were determined in situ by means of soil sampling and infiltration tests.

### Table 2. Soil Hydraulic Properties of Soil and Relative Variations

<table>
<thead>
<tr>
<th>Initial Value</th>
<th>Percentage</th>
<th>( \theta_r ) (cm(^3)cm(^{-3}))</th>
<th>( \theta_s ) (cm(^3)cm(^{-3}))</th>
<th>( K_s ) (cm d(^{-1}))</th>
<th>( n ) (dimensionless)</th>
<th>( \alpha ) (cm(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.015</td>
<td>0.415</td>
<td>0.541</td>
<td>1.423</td>
<td>0.014</td>
</tr>
<tr>
<td>Decrease</td>
<td>10</td>
<td>0.013</td>
<td>0.373</td>
<td>0.487</td>
<td>1.280</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.012</td>
<td>0.332</td>
<td>0.433</td>
<td>1.138</td>
<td>0.011</td>
</tr>
<tr>
<td>Increase</td>
<td>10</td>
<td>0.016</td>
<td>0.456</td>
<td>0.595</td>
<td>1.565</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.018</td>
<td>0.498</td>
<td>0.649</td>
<td>1.707</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Note: \( a = \) empirical parameter; \( K_s = \) saturated hydraulic conductivity; \( qr = \) residual humidity; \( qs = \) saturated humidity; \( n = \) empirical parameter.

The sensitivity of the model was analyzed based on the variations in the modeling of the matric potential and soil moisture profiles. To facilitate the comparison of sensitivity for each of the parameters, the relative sensitivity (\( Sr \)) was used. \( Sr \) is defined by the ratio of the sum of the values of the model response (\( \Delta S \)) to a small variation in the sum of the input values (\( \Delta E \)), normalized by the initial output (\( S_i \)) and input (\( E_i \)) values (McCuen and Snyder, 1986) (Equation 4).
According to Chaves (2009), Sr values above 1.5 represent a high level of sensitivity for the parameter and values below 0.5 represent a low sensitivity for the parameter. Sr values between 0.5 and 1.5 indicate that the model has an intermediate sensitivity for the given parameter.

**Results and Discussion**

The relative sensitivity (Sr) values for the increments and decrements of each parameter can be seen in Table 3.

The qr parameter had the lowest relative sensitivity value among the moisture values, with a Sr = 0.06 for decreases of 20%. On the other hand, saturated volumetric moisture was the parameter that generated the highest relative sensitivity (Sr = 2.16) in the Hydrus-1D model, when simulating soil water dynamics. The matric potential values generated from the 10% increase in the parameter a resulted in intermediate relative sensitivities, Sr = 0.5.

<table>
<thead>
<tr>
<th>Parameter/increase</th>
<th>α (cm⁻¹)</th>
<th>Ks (cm d⁻¹)</th>
<th>qr (cm³ cm⁻³)</th>
<th>qs (cm³ cm⁻³)</th>
<th>n (dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10%</td>
<td>h 0.41</td>
<td>q 0.44</td>
<td>H 0.47</td>
<td>q 0.16</td>
<td>h 0.45</td>
</tr>
<tr>
<td>-20%</td>
<td>h 0.18</td>
<td>q 0.38</td>
<td>H 0.25</td>
<td>q 0.08</td>
<td>h 0.23</td>
</tr>
<tr>
<td>+10%</td>
<td>h 0.50</td>
<td>q 0.10</td>
<td>H 0.44</td>
<td>q 0.15</td>
<td>h 0.45</td>
</tr>
<tr>
<td>+20%</td>
<td>h 0.27</td>
<td>q 0.17</td>
<td>H 0.21</td>
<td>q 0.07</td>
<td>h 0.23</td>
</tr>
</tbody>
</table>

*Note: h = matric potential; q = soil moisture; α = empirical parameter; Ks = saturated hydraulic conductivity; qr = residual humidity; qs = saturated humidity; n = empirical parameter.*

For the values of the matrix potential generated from the variations of 10 and 20% of the parameter Ks, the model presented a low level of sensitivity (Sr <0.5). However, soil moisture values for all variations of saturated hydraulic conductivity showed that Hydrus-1D was also not very sensitive. As was the case for Ks, the variations of the parameter qr also led the model to have low relative sensitivity values (Sr <0.5), both for the matrix potential and for the soil moisture (Table 3).

For the values of the matrix potential generated from the 10 and 20% variations of the parameter Ks, the model presented a low level of sensitivity (Sr <0.5). Soil moisture values for all variations of saturated hydraulic conductivity also showed the Hydrus-1D model to not be very sensitive.
As was the case for Ks, variations in the parameter qr also produced low relative sensitivity values (Sr <0.5), both for the matric potential and for soil moisture (Table 3).

For the parameter n of the Van Genuchten-Mualem equation, the values of the matric potential obtained with Hydrus-1D presented low sensitivity for decreases of 10 and 20% and an increase of 20% (Sr <0.5). However, at an increase of 10%, the sensitivity of matric potential was high, Sr = 1.61. As for moisture, 10 and 20% decreases showed the model to have an intermediate sensitivity, and increases of 10 and 20% resulted in low sensitivity.

The matric potential curves generated from the percentage changes in the value of parameter a (Figure 1a) show that even with significant variations, there was very little discrepancy in the results obtained. For the moisture values, however, the curves separated by a greater distance (Figure 1b), demonstrating a certain imprecision in the model, even with low relative sensitivity.

![Figure 1](image_url). Modeling of the matric potential (a) and soil moisture (b), at 0.05 m depth, based on the variation of parameter a.

Both the matric and moisture curves (Figure 2a and 2b) simulated from variations in Ks show that the model’s sensitivity to this parameter is low, demonstrated by the overlap of the curves. Compared to the results shown in Table 3, it can be inferred that variations of this parameter do not cause Hydrus-1D to generate significantly imprecise results.

Confirming the relative sensitivity values (Table 3), Figures 3a and 3b demonstrate that Hydrus-1D was not very sensitive to variations in qr. This is shown by the close proximity of the curves (overlap), similar to the moisture and matric potential curves generated by variations of the parameter Ks.
Figure 2. Modeling of the matric potential (a) and soil moisture (b), at 0.05 m depth, based on the variation of parameter Ks.

Figure 3. Modeling of the matric potential (a) and soil moisture (b), at 0.05 m depth, based on the variation of parameter qr.
The matric potential curves for variations of n (Figure 4a) demonstrate that the sensitivity of the model is low for most parameter variations. However, the model was highly sensitive for 10% increases, shown by the distance between the n + 10% and n curves.

The moisture curves (Figure 4b) show a distance of up to 0.09 cm$^3$cm$^{-3}$ between the n-10% and n-20% curves and the n curve. Thus, Hydrus-1D has an intermediate to high sensitivity, partially validating the values presented in Table 3.

With regard to the matric potential curves referring to variations in saturated water content (Figure 5a), the sensitivity of model was low for any increase or decrease in the parameter. For soil moisture values, however, Hydrus-1D sensitivity was intermediate for increases and decreases of 10% and high for increments and decreases of 20% (Figure 5b).

These inferences are derived from the distance between the moisture curves of q$_s$ ± 20% and q$_s$ of up to 0.075 cm$^3$cm$^{-3}$. For the moisture curves of q$_s$ ± 10% and q$_s$, this distance reaches up to 0.03 cm$^3$cm$^{-3}$ from the parameter q$_s$ curve. Thus, the moisture curves for the q$_s$ variations are consistent with the relative sensitivity results shown in Table 3.

In some studies, sensitivity analysis also showed distortions of the data estimated using the Hydrus-1D model (Cheviron; Coquet, 2009; Vázquez et al., 2013, Chen et al., 2014; Pfletschinger et al., 2014; Šimůnek et al., 2016).
Figure 5. Modeling of the matric potential (a) and soil moisture (b), at 0.05 m depth, based on the variation of parameter $q_s$.

In order to test the sensitivity of Hydrus-1D to hydrodynamic parameters of differently-textured soil using relative sensitivity, Chaves (2009) concluded that the model was highly sensitive to parameter $n$ ($Sr = 2.79$) and intermediately sensitive to $q_s$, $a$, and $K_s$, with values of 1.01, 0.85, and 0.83, respectively. It showed low sensitivity for $q_r$ ($Sr = 0.30$).

Although simulated for different types of soils, the results obtained in this study corroborate those found by Chaves (2009), as the model was relatively insensitive to changes in parameter $q_r$ in both studies.

When performing sensitivity analyses on two types of soils (Orthic Luvisols and Eutric Cambisol), Cheviron & Coquet (2009) verified that Hydrus-1D is highly sensitive to variations in parameter $q_s$. In addition, they also found that variations related to parameter $n$ have significant impacts on the sensitivity of the model. Thus, even though the soil studied by these authors was different, the results are similar to those obtained in this research.

Pfletschinger et al. (2014), studying groundwater in arid soils, concluded that residual moisture ($q_r$), and saturated hydraulic conductivity ($K_s$) were the most relevant parameters in the characterization of ground water flow patterns. According to Chen et al. (2012), this shows that Hydrus-1D may respond differently for different types of soils, since the results obtained by Pfletschinger et al. (2014) are different than those found in this study.
Although the relative sensitivity of the model was low or intermediate for some parameters, it was verified that increments and decrements in the moisture value of parameter a generated values overestimated by 8.7%. Increased and decreased values of $q_s$ produced results overestimated by up to 20%. For the $n$ parameter of the van Genuchten-Mualem equation, the model showed that decreases generated variation of up to 32% and increases of up to 21%. However, the other parameters did not show significant variations.

The results obtained for the parameter $a$ resemble those presented by Soares et al. (2016) using the SiSPAT program, which found that the 10 and 20% percent increments and decrements generated overestimated and underestimated results of up to 7%. For $K_s$, the model used by these authors presented similar results to those obtained in the present study, with responses very close to the reference values. However, for parameter $n$, SiSPAT generated discrepancies of approximately 50% of the reference value, making it considerably more sensitive than Hydrus-1D.

**Conclusions**

The matric potential values proved to be highly sensitive to variations in the parameter $n$ with the use of Hydrus-1D, resulting in a relative sensitivity of 1.61, over increments of 10%.

Saturated volumetric moisture was the parameter displaying the highest level of relative sensitivity in the model when simulating soil water dynamics. An increase of 20% triggered a high relative sensitivity ($Sr = 2.16$), and produced an intermediate relative sensitivity for saturated moisture values.

For all other parameters ($a$, $K_s$ and $q_r$), the model’s relative sensitivity was low for all of the increases and decreases analyzed. Therefore, the parameter $q_r$ can be considered to be null, because it has negligible influence on the modeling of matric potential and soil moisture.

The results obtained in this study are valid only for the Yellow Latosol studied, and inferences should not be generalized, as soil type is one of the factors that can interfere in the sensitivity of the model. Sensitivity analysis should therefore be performed whenever possible to ensure the accuracy and reliability of the results.

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